Investment Process

Incorporation of other constraints/assumptions

We can other constraints like depending on the practical situation a person faces.

Few examples are:

$$\sum_{i=1}^{n} w_i d_i \geq D^*$$

$$w_i^{\max} \geq w_i \geq w_i^{\min}$$

Investment Process

Alternative definition of SS (Lintner's definition)

As SS involves putting an amount of money equal to SS (for security reasons), thus SS is use of fund rather than a source of fund (as we have seen till now). The total fund that the broker-dealer invests short, plus the fund invested long, must add up to the original value of the investment. Since for SS $w_i \leq 0$, the proportion of the fund invested in the SS is $|w_i|$. Thus we have to use an important condition which is

$$\sum_{i=1}^{n} | w_i | = 1$$

Investment Process

The two important points about which we should be concerned while implementing the portfolio theory are

- Simplification of the amount and type of input data needed to perform portfolio analysis
- Simplification of the computational procedure needed to calculate the optimal portfolio

Investment Process

To solve the above mentioned problems we can use

- Index models
  - Single index model
  - Multi index model
- Averaging techniques

Investment Process

Single index models

Under this method the most important assumption is that the co-movement between stocks is due to a single common influence or index (which we will later call as the market index). These models are not only used to estimate the correlation matrix, but also used in efficient market tests and in equilibrium tests

Investment Process

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i$$

Assumptions

$E[\varepsilon_i] = 0$ $\forall$ i=1,2,…..,n

$E[\varepsilon_i (R_m - \bar{R}_m)] = 0$ $\forall$ i=1,2,…..,n

$E[\varepsilon_i \varepsilon_j] = 0$ $\forall$ i≠j, i,j=1,2,…..,n

$V[\varepsilon_i] = \sigma^2_{\varepsilon(i)}$ $\forall$ i=1,2,…..,n

$V[R_m] = \sigma^2_m$

Investment Process

Results

1) $\overline{R}_i = \alpha_i + \beta_i \overline{R}_m$ $\forall$ i=1,2,…..,n

2) $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon(i)}^2$ $\forall$ i=1,2,…..,n

3) $\sigma_{i,j} = \beta_i \beta_j \sigma_m^2$ $\forall$ i≠j, i,j=1,2,……,n

Investment Process

Assume α = 3 and β = 2.0

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i$$

Month	Return(i)	Return(m)	
1	14	5	2*5
2	10	4	2*4
3	25	10	2*10
4	16	7	2*7
5	4	1	2*1

Hence the errors are: +1, -1, +2, -1, -1 which adds up to 0 as the case should be

Investment Process

Results

$$\bar{R}_P = \sum_{i=1}^{n} w_i \alpha_i + \sum_{i=1}^{n} w_i \beta_i \bar{R}_m$$

$$\sigma_P^2 = \sum_{i=1}^{n} w_i^2 \beta_i^2 \sigma_m^2 + \sum_{i=1}^{n} w_i \sigma_{\varepsilon(i)}^2 + \sum_{i=1}^{n} \sum_{j=1 (i \neq j)}^{n} w_i w_j \beta_i \beta_j \sigma_m^2$$

$$\beta_P = \sum_{i=1}^{n} w_i \beta_i$$

$$\alpha_P = \sum_{i=1}^{n} w_i \alpha_i$$

Investment Process

$$\overline{R}_P = \alpha_P + \beta_P \overline{R}_m$$

Thus if the portfolio P is taken to be the market portfolio (i.e., all stocks held in the same proportion as they are in R_m), then the expected return on P must be $\overline{R}_m$

Investment Process

Risk of the portfolio

$$\sigma_P^2 = \sum_{i=1}^{n} w_i^2 \beta_i^2 \sigma_m^2 + \sum_{i=1}^{n} w_i^2 \sigma_{\varepsilon(i)}^2 + \sum_{i=1}^{n} \sum_{j=1(i \neq j)}^{n} w_i w_j \beta_i \beta_j \sigma_m^2$$

$$\sigma_P^2 = \beta_P^2 \sigma_m^2 + \sum_{i=1}^{n} w_i^2 \sigma_{\varepsilon(i)}^2$$

Investment Process

From this equation (i.e., the variance of the portfolio) we can have some feel of how diversification takes place. Assume for the time being that we invest in equal proportions in the ′n′ number of stocks, then $\sigma_P^2 = \beta_P^2 \sigma_m^2 + \frac{1}{n^2} \sum_{i=1}^{n} \sigma_{\varepsilon(i)}^2$

Investment Process

For the risk $\sigma_P^2 = \beta_P^2 \sigma_m^2 + \sum_{i=1}^{n} w_i^2 \sigma_{\varepsilon(i)}^2$ of any portfolio we have the first term known as **non-diversifiable/market/systematic** risk. While the second term is the **diversifiable/non-market/nonsystematic** risk

Investment Process

For the stock prices/indices (assuming they are log-normally distributed) we have $r=\ln(SP_2/SP_1)$. In place of R we can use r and do all the calculation we have done so far.

Investment Process

- In India we have the BSE and the NSE
- The most well known indices are BSE30 [http://www.bseindia.com] and S&P CNX Nifty [http://www.nseindia.com]
- The BSE30 has 30 companies, while S&P CNX Nifty has 50 companies as the constituent scripts in the portfolio/basket

Investment Process

BSE30 companies (as on end of January 2009)

ACC Ltd.
Bharat Heavy Electricals Ltd.
Bharti Airtel Ltd.
DLF Ltd.
Grasim Industries Ltd.
HDFC
HDFC Bank Ltd.
Hindalco Industries Ltd.
Hindustan Unilever Ltd.
ICICI Bank Ltd.
Infosys Technologies Ltd.
ITC Ltd.
Jaiprakash Associates Ltd.
Larsen & Toubro Limited
Mahindra & Mahindra Ltd.
Maruti Suzuki India Ltd.
NTPC Ltd.
ONGC Ltd.
Ranbaxy Laboratories Ltd.
Reliance Communications Ltd.
Reliance Industries Ltd.
Reliance Infrastructure Ltd.
State Bank of India
Sterlite Industries (India) Ltd.
Sun Pharmaceutical Industries Ltd.
Tata Consultancy Services Ltd.
Tata Motors Ltd.
Tata Power Company Ltd.
Tata Steel Ltd.
Wipro Ltd.

Investment Process

S&P CNX Nifty

ABB Ltd.	ACC Ltd.	Ambuja Cements Ltd.	BHEL
BPCL	Bharti Airtel Ltd.	Cairn India Ltd.	Cipla Ltd.
DLF Ltd.	GAIL	Grasim Industries	HCL Technologies
HDFC Bank	Hero Honda Motors	Hindalco Industries	HUL
HDFC	I T C Ltd.	ICICI Bank	Idea Cellular
Infosys	L&T	M&M	Maruti
NTPC Ltd.	NALCO	ONGC	PNB
Ranbaxy	Reliance Capital	Unitech	Wipro
Reliance Industries	Reliance Infrastructure	Reliance Petroleum	Reliance Power Ltd.
Siemens	SBI	SAIL	Sterlite Industries
Suzlon Energy Ltd.	Tata Communications	TCS	Tata Motors
Tata Power	TISCO	ZEE Entertainment	Reliance Communications
Power Grid Corporation of India			
Sun Pharmaceutical Industries Ltd.			

Investment Process

- Draw the scatter plot of S&P CNX Nifty$_t$ vs. S&P CNX Nifty$_{t-1}$
- For each of the script draw the graph of script price with S&P CNX Nifty
- For each of the script draw the graph of script return with S&P CNX Nifty return
- For each script draw the volatility of the script price with volatility of S&P CNX Nifty
- For each script draw the volatility of the script return with volatility of S&P CNX Nifty return
- For each script draw the total volumes traded against the day
- For each script draw the total value traded against the day

Investment Process

S&P CNX Nifty Price

Date	Open	High	Low	Close
03-Jul-2006	3128.75	3160.35	3114.85	3150.95
04-Jul-2006	3151.05	3177.40	3130.30	3138.65
05-Jul-2006	3136.95	3201.20	3121.80	3197.10
06-Jul-2006	3197.50	3197.50	3138.40	3156.40
07-Jul-2006	3157.95	3193.10	3056.30	3075.85
10-Jul-2006	3077.10	3147.70	3064.10	3142.00
11-Jul-2006	3145.30	3146.00	3100.40	3116.15
12-Jul-2006	3124.95	3201.35	3078.25	3195.90
13-Jul-2006	3196.30	3208.85	3148.95	3169.30
14-Jul-2006	3166.25	3166.25	3089.55	3123.35
17-Jul-2006	3123.65	3125.10	2999.35	3007.55

Investment Process

S&P CNX Nifty price returns

Date	Close	Rate=$\ln(P_t/P_{t-1})$
03-Jul-2006	3150.95	
04-Jul-2006	3138.65	-0.0039
05-Jul-2006	3197.10	+0.0185
06-Jul-2006	3156.40	-0.0128
07-Jul-2006	3075.85	-0.0259
10-Jul-2006	3142.00	+0.0213
11-Jul-2006	3116.15	-0.0083
12-Jul-2006	3195.90	+0.0253
13-Jul-2006	3169.30	-0.0083
14-Jul-2006	3123.35	-0.0146
17-Jul-2006	3007.55	-0.0378

Investment Process

Problem # 4

Month	A	B	C	S&P
1	12.05	25.20	31.67	12.28
2	15.27	2.86	15.82	5.99
3	-4.12	5.45	10.58	2.41
4	1.57	4.56	-14.43	4.48
5	3.16	3.72	31.98	4.41
6	-2.79	10.79	-0.72	4.43
7	-8.97	5.38	-19.64	-6.77
8	-1.18	-2.97	-10.00	-2.11
9	1.07	1.52	5.63	6.16
10	12.75	10.75	-4.67	2.47
11	7.48	3.79	-4.67	2.47
12	-0.94	1.32	7.94	-1.15

Given the above data find the mean return and variance for each stock

Investment Process

Problem # 5

For the **five (5)** individual scripts you have taken find their respective returns for the time period 01-01-2006 to 30-06-2006. Also find the corresponding S&P CNX Nifty returns.

Calculate the respective values of β_i's for the five companies considering S&P CNX Nifty as the market index.

Investment Process

Problem # 6

For **five (5)** individual scripts taken from NYSE find their respective returns for the time period 01-01-2006 to 30-06-2006. Also find the corresponding NYSE (DJIA) returns.

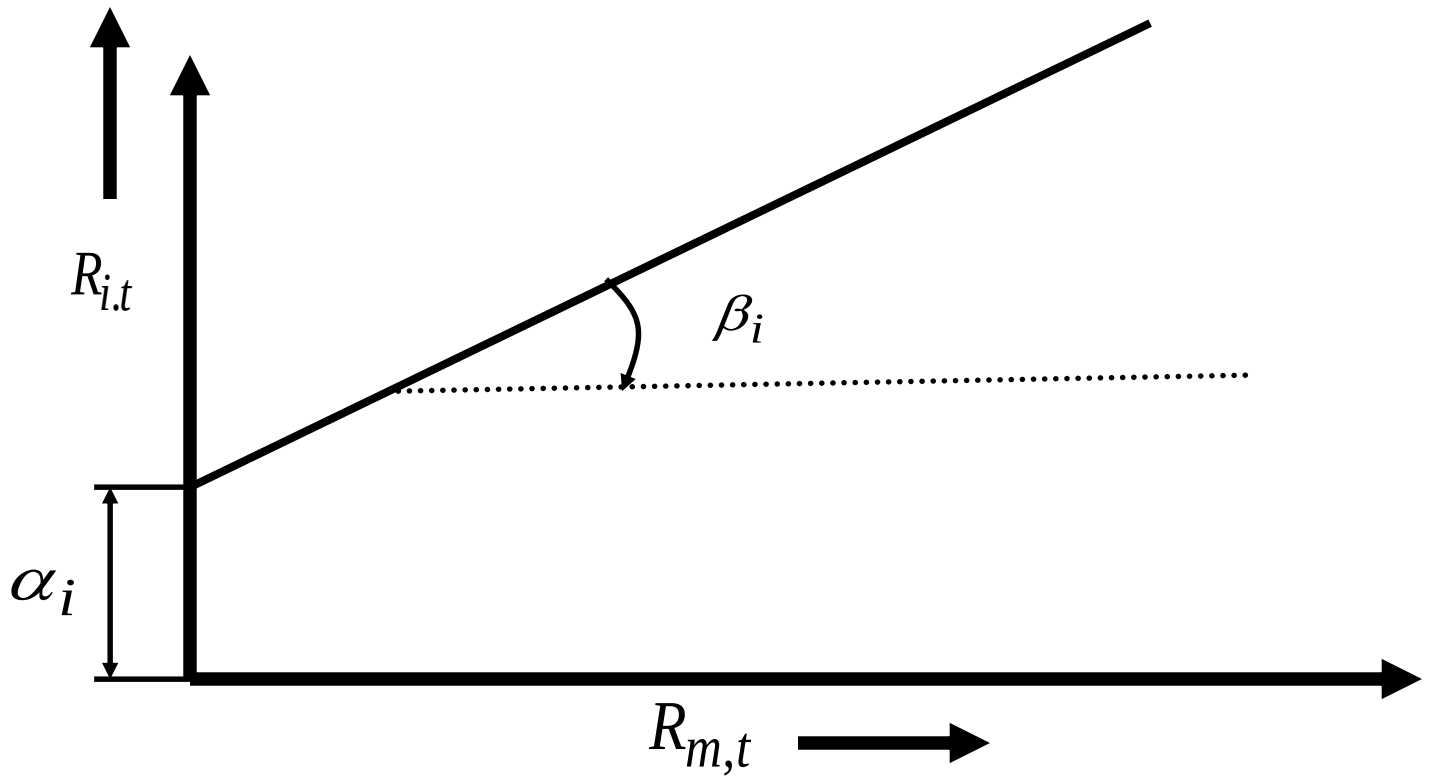
Calculate the respective values of β_i′s for the five companies considering DJIA as the market index. For this you need to get the US 91 days T-bill value to find the corresponding risk free interest rate pertaining to USA.

Investment Process

The next question is how do we estimate the values of betas. We can use the historical beta values as an estimate of the future beta values.

The equation $R_i = \alpha_i + \beta_i R_m + \varepsilon_i$ is generally not the same with respect to time, in the sense the values of α_i, $\sigma^2_{\varepsilon(i)}$ and β_i all change with respect to time. In case we consider them to be independent with respect to time then we have some straight forward procedure for calculating these values.

Investment Process



Investment Process

$$\beta_i = \frac{\sigma_{i,m}}{\sigma_m^2} = \frac{\sum_{t=1}^{T}[(R_{i,t} - \bar{R}_{i,t})(R_{m,t} - \bar{R}_{m,t})]}{\sum_{t=1}^{T}(R_{m,t} - \bar{R}_{m,t})^2}$$

$$\alpha_i = \bar{R}_{i,t} - \beta_i \bar{R}_{m,t}$$

$$\sigma_{\varepsilon(i)}^2 = \frac{1}{T}\sum_{t=1}^{T}[R_{i,t} - (\alpha_i + \beta_i R_{m,t})]^2$$

Investment Process

We may also be interested in determining the following coefficient which is $\rho_{i,m} = \frac{\sigma_{i,m}}{\sigma_i \sigma_m} = \beta_i \frac{\sigma_m}{\sigma_i}$

Remember for all the calculation above we have assumed α_i, $\sigma^2_{\varepsilon(i)}$ and β_i are <u>independent</u> of time. Having said that, values we calculate using the above three equations gives us an estimate of the true values of α_i, $\sigma^2_{\varepsilon(i)}$ and β_i and they may be subject to errors. In general these calculated values are not equal to their respective true values.

Investment Process

Multi index models

Using this methodology we try to capture some of the non-market influences that cause securities to move together. Searching for the non-market influences is akin to searching for corresponding economic factors/influences or structural groups (industries) that account for common movement in stock prices, beyond that accounted for by the market index itself. This model introduces extra indices with the hope of capturing additional information.

Investment Process

The basic use of multi index model is to predict the correlation coefficients. Some other uses are:

- To form expectations about returns and study the change in return due to impact of events.
- Method of tailoring the return distribution of a portfolio to the specific needs of the investor.
- Method of attributing the cause of good or bad performance of a portfolio.

Investment Process

$$R_i = \alpha_i + b_{i(1)} I_1 + b_{i(2)} I_2 + ... + b_{i(L)} I_L + \varepsilon_i$$

Assumptions

$E[\varepsilon_i] = 0$	$\forall$ i=1,...,n
$V[\varepsilon_i] = \sigma^2_{\varepsilon(i)}$	$\forall$ i=1,...,n
$V[I_j] = \sigma^2_{I(j)}$	$\forall$ j=1,…,L
$E[(I_j - \bar{I}_j)(I_k - \bar{I}_k)] = 0$	$\forall$ j≠k, j,k=1,...,L
$E[(\varepsilon_i - \bar{\varepsilon}_i)(I_j - \bar{I}_j)] = 0$	$\forall$ i=1,...,n & j=1,…,L

Investment Process

$$\bar{R}_i = \alpha_i + b_{i(1)}\bar{I}_1 + b_{i(2)}\bar{I}_2 + ... + b_{i(L)}\bar{I}_L$$

$$\sigma_i^2 = b_{i(1)}^2\sigma_{I(1)}^2 + b_{i(2)}^2\sigma_{I(2)}^2 + ... + b_{i(L)}^2\sigma_{I(L)}^2 + \sigma_{\varepsilon(i)}^2$$

$$\sigma_{i,j} = b_{i(1)}b_{j(1)}\sigma_{I(1)}^2 + b_{i(2)}b_{j(2)}\sigma_{I(2)}^2 + ... + b_{i(L)}b_{j(L)}\sigma_{I(L)}^2 + \sigma_{I(L)}^2$$

Investment Process

Some variables which are used in multi index model are (this is not an exhaustive list)

1) Economic growth
2) Business cycle
3) Long-term interest rates
4) Short term interest rates
5) Inflation, i.e., CPI
6) Foreign currency rates, like Rs. vs US$, Rs. vs Yen, Rs. vs UK Pound, etc.

Investment Process

Problem # 7

Prepare a 3 page page (A4 size) write up about how multi index models are implemented in the industry. Follow the norm of font size 11, Times New Roman, margins of 1” on all sides and paragraph space of 1 unit. Do a thorough search on the net or in the book and do a comprehensive work.

Investment Process

Problem # 8

Taking the index of **one** company (01-01-2005 – 30-06-2006) model a multi index model. You are free to choose the variables which you think are appropriate. This is just a rudimentary exercise to check the working principle of the multi index model. This work may not yield you an exact answer but would give you an idea about the working principle of multi index model.

Investment Process

Average techniques

Average (smoothing) of the historical correlation data as a forecast of the future. Note for multi normal distribution we can assume that S is the best estimate of Σ.

Investment Process

A better averaging model would be to assume that there was a common mean correlation within and between groups of stocks. This is to say that if we have one group as steel stocks and the other as cement stocks, then the past correlation structure between the steel stocks would be the best predictor of the future correlation structure between steel stocks. Similarly for cement stocks. Moreover the past correlation between the steel stocks group and the cement stocks group would also be the best predictor of the future correlation structure between the steel stocks group and cement stocks group.

Investment Process

Moreover the correlation between any steel stock and any cement stock is assumed to be same. To make this calculation simple we take this value to the average of all the correlations between each steel and cement stock taken in pairs.

Investment Process

Simple techniques for determining the efficient frontier

We will describe here the methods for selecting the optimal portfolio considering

1) Single index model
2) Constant variance-covariance matrix

Investment Process

Using the single index model

We would be interested in finding a single number that measures the desirability of including a stock in the optimal portfolio. The desirability of any stock to be included in the portfolio depends on the excess return to beta ratio $\frac{\overline{R}_i - R_f}{\beta_i}$

This means the additional return over and above the risk free interest rate divided by the risk (in this case the beta of the stock)

Investment Process

Now if there is no SS then if this ratio is –ve or zero we would not include the particular stock in the portfolio. For SS if this ratio is –ve it means that we SS this stock, i.e., borrow this stock so that it is included in our portfolio.

Investment Process

Without SS

$$\frac{\overline{R}_i - R_f}{\beta_i} > C^*$$ include

$$\frac{\overline{R}_i - R_f}{\beta_i} < C^*$$ do not include

With SS

$$\frac{\overline{R}_i - R_f}{\beta_i} > C^*$$ include

$$\frac{\overline{R}_i - R_f}{\beta_i} < C^*$$ include with SS

Investment Process

The problem has the condition

- SS being not allowed
- Risk less lending and borrowing being allowed

Investment Process

The rule works as follows

- Find the excess return to beta ratio for each stock under consideration and rank them from the highest to the lowest
- The optimal consists of investing in all stocks with $\frac{\overline{R}_i - R_f}{\beta_i} > C^*$
- And not investing in any stock with

$$\frac{\overline{R}_i - R_f}{\beta_i} \leq C^*$$

Investment Process

We assume $R_f = 5\%$ and $\sigma_m^2 = 10\%$

i	$\overline{R}_i$	$\overline{R}_i - R_f$	β_i	$\sigma^2_{\varepsilon(i)}$	$\frac{\overline{R}_i - R_f}{\beta_i}$
1	15	10	1	50	10
2	17	12	1.5	40	8
3	12	7	1	20	7
4	17	12	2	10	6
5	11	6	1	40	6
6	11	6	1.5	30	4
7	11	6	2	40	3
8	7	2	0.8	16	2.5
9	7	2	1	20	2
10	5.6	0.6	0.6	6	1

Investment Process

i	$\frac{\bar{R}_i - R_f}{\beta_i}$	$(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(i)}})\beta_i$	$\frac{\beta_i^2}{\sigma^2_{\varepsilon(i)}}$	$\sum_{j=1}^{i} (\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(j)}})\beta_j$	$\sum_{j=1}^{i} \frac{\beta_j^2}{\sigma^2_{\varepsilon(j)}}$
1	10	2/10	2/100	2/10	2/100
2	8	4.5/10	5.63/100	6.5/10	7.63/100
3	7	3.5/10	5/100	10/10	12.63/100
4	6	24/10	40/100	34/10	52.63/100
5	6	1.5/10	2.5/100	35.5/10	55.13/100
6	4	3/10	7.5/100	38.5/10	62.63/100
7	3	3/10	10/100	41.5/10	72.63/100
8	2.5	1/10	4/100	42.5/10	76.63/100
9	2	1/10	5/100	43.5/10	81.63/100
10	1	0.6/10	6/100	44.1/10	87.63/100

Investment Process

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^{i} \left(\frac{\bar{R}_i - R_f}{\sigma_{\varepsilon(j)}^2}\right)\beta_j}{1 + \sigma_m^2 \sum_{j=1}^{i} \frac{\beta_j^2}{\sigma_{\varepsilon(j)}^2}}$$

i	
1	1.67
2	3.69
3	4.42
4	5.43
5	**5.45**
6	5.30
7	5.02
8	4.91
9	4.75
10	4.52

Investment Process

This C* implies that for all assets above (in the sense $\frac{\bar{R}_i - R_f}{\beta_i} > C^*$)

will be included in the portfolio. And for assets with $\frac{\bar{R}_i - R_f}{\beta_i} < C^*$ they

would not be included. This is due to the fact that SS is not being allowed

Investment Process

Remember one important thing. The summation is being done from j to i. This is true for this case when SS is not being allowed

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^{i} \left(\frac{\bar{R}_i - R_f}{\sigma_{\varepsilon(j)}^2}\right)\beta_j}{1 + \sigma_m^2 \sum_{j=1}^{i} \frac{\beta_j^2}{\sigma_{\varepsilon(j)}^2}}$$

Investment Process

$$w_i = \frac{\beta_i}{\sigma^2_{\varepsilon(i)}}\left[\frac{\bar{R}_i - R_f}{\beta_i} - C^*\right]$$

$$C^* = \sigma_m^2 \sum_{j=1}^{n} \beta_j w_j = \frac{\sigma_m^2 \sum_{j=1}^{n} \left(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(j)}}\right)\beta_j}{1 + \sigma_m^2 \sum_{j=1}^{n} \frac{\beta_j^2}{\sigma^2_{\varepsilon(j)}}}$$

The n here does not mean the whole set of assets given to you, but only the set of assets which you have included in your portfolio. Here in this example it is 5.

Investment Process

$$w_i = \frac{\beta_i}{\sigma^2_{\varepsilon(i)}}\left[\frac{\bar{R}_i - R_f}{\beta_i} - \frac{\sigma^2_m \sum_{j=1}^{n} \left(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(j)}}\right)\beta_j}{1 + \sigma^2_m \sum_{j=1}^{n} \frac{\beta_j^2}{\sigma^2_{\varepsilon(j)}}}\right]$$

i	w(i) (non-normalized wghts)	w*(i) (normalized wghts)
1	0.166667	0.284769
2	0.161702	0.276287
3	0.129006	0.220421
4	0.114172	0.195075
5	0.013724	0.023448
6		
7		
8		
9		
10		

Investment Process

The problem has the condition

- SS being allowed (normal definition)
- Risk less lending and borrowing being allowed

Investment Process

The rule works as follows

- Find the excess return to beta ratio for each stock under consideration and rank them from the highest to the lowest
- The optimal consists of investing in all stocks with $\frac{\overline{R}_i - R_f}{\beta_i} > C^*$
- And also investing in stocks (SS) with

$$\frac{\overline{R}_i - R_f}{\beta_i} < C^*$$

Investment Process

We assume $R_f = 5\%$ and $\sigma_m^2 = 10\%$

i	$\overline{R}_i$	$\overline{R}_i - R_f$	β_i	$\sigma^2_{\varepsilon(i)}$	$\frac{\overline{R}_i - R_f}{\beta_i}$
1	15	10	1	50	10
2	17	12	1.5	40	8
3	12	7	1	20	7
4	17	12	2	10	6
5	11	6	1	40	6
6	11	6	1.5	30	4
7	11	6	2	40	3
8	7	2	0.8	16	2.5
9	7	2	1	20	2
10	5.6	0.6	0.6	6	1

Investment Process

i	$\frac{\bar{R}_i - R_f}{\beta_i}$	$(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(i)}})\beta_i$	$\frac{\beta_i^2}{\sigma^2_{\varepsilon(i)}}$	$\sum_{j=1}^{n}(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(j)}})\beta_j$	$\sum_{j=1}^{n}\frac{\beta_j^2}{\sigma^2_{\varepsilon(j)}}$
1	10	2/10	2/100	44.1/10	87.63/100
2	8	4.5/10	5.63/100	44.1/10	87.63/100
3	7	3.5/10	5/100	44.1/10	87.63/100
4	6	24/10	40/100	44.1/10	87.63/100
5	6	1.5/10	2.5/100	44.1/10	87.63/100
6	4	3/10	7.5/100	44.1/10	87.63/100
7	3	3/10	10/100	44.1/10	87.63/100
8	2.5	1/10	4/100	44.1/10	87.63/100
9	2	1/10	5/100	44.1/10	87.63/100
10	1	0.6/10	6/100	44.1/10	87.63/100

Investment Process

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{n} \left(\frac{\bar{R}_i - R_f}{\sigma_{\varepsilon(i)}^2}\right)\beta_i}{1 + \sigma_m^2 \sum_{i=1}^{n} \frac{\beta_i^2}{\sigma_{\varepsilon(i)}^2}}$$

i	
1	4.52
2	4.52
3	4.52
4	4.52
5	4.52
6	4.52
7	4.52
8	4.52
9	4.52
10	4.52

Investment Process

Now there is no C* which is maximum. As we are considering all the assets (due to SS), hence we have only one value (which we, for our convenience will also denote here by C*).

Assets with $C_i > C^*$ will be included in the portfolio and for those with $C_i < C^*$ would be included considering SS

Investment Process

There is no C* which is maximum. We are considering all the assets (due to SS), hence we have only one value (which we, for our convenience will also denote here by C*). Assets with

$$\frac{\overline{R}_i - R_f}{\beta_i} > C^*$$

will be included in the portfolio and for those $\frac{\overline{R}_i - R_f}{\beta_i} < C^*$

with would be included considering SS

Investment Process

Remember one important thing. The summation is being done from j to n (i.e., all assets you have been provided with).

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{n} \left(\frac{\bar{R}_i - R_f}{\sigma_{\varepsilon(i)}^2}\right)\beta_i}{1 + \sigma_m^2 \sum_{i=1}^{n} \frac{\beta_i^2}{\sigma_{\varepsilon(i)}^2}}$$

Investment Process

$$w_i = \frac{\beta_i}{\sigma^2_{\varepsilon(i)}}\left[\frac{\bar{R}_i - R_f}{\beta_i} - C^*\right]$$

$$C^* = \sigma^2_m \sum_{i=1}^{n} \beta_i wi = \frac{\sigma^2_m \sum_{i=1}^{n} \left(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(i)}}\right)\beta_i}{1 + \sigma^2_m \sum_{i=1}^{n} \frac{\beta_i^2}{\sigma^2_{\varepsilon(i)}}}$$

The n here means the whole set of assets given to you. Here in this example it is 10.

Investment Process

$$w_i = \frac{\beta_i}{\sigma^2_{\varepsilon(i)}}\left[\frac{\bar{R}_i - R_f}{\beta_i} - \frac{\sigma^2_m \sum_{i=1}^{n}\left(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(i)}}\right)\beta_i}{1+\sigma^2_m \sum_{i=1}^{n}\frac{\beta_i^2}{\sigma^2_{\varepsilon(i)}}}\right]$$

i	w(i) (non-normalized wghts)	w*(i) (normalized wghts)
1	0.10965	6.15451
2	0.13060	7.33022
3	0.12414	6.96730
4	0.29654	16.64391
5	0.03707	2.08049
6	-0.02586	-1.45167
7	-0.07586	-4.25799
8	-0.10086	-5.66116
9	-0.12586	-7.06432
10	-0.35173	-19.74129

Investment Process

The problem has the condition

- SS being allowed (Lintner's definition)
- Risk less lending and borrowing being allowed

Investment Process

The rule works as follows

- Find the excess return to beta ratio for each stock under consideration and rank them from the highest to the lowest
- The optimal consists of investing in all stocks with $\frac{\bar{R}_i - R_f}{\beta_i} > C^*$
- And also investing in stocks (SS) with

$$\frac{\bar{R}_i - R_f}{\beta_i} < C^*$$

Investment Process

We assume $R_f = 5\%$ and $\sigma_m^2 = 10\%$

i	$\overline{R}_i$	$\overline{R}_i - R_f$	β_i	$\sigma^2_{\varepsilon(i)}$	$\frac{\overline{R}_i - R_f}{\beta_i}$
1	15	10	1	50	10
2	17	12	1.5	40	8
3	12	7	1	20	7
4	17	12	2	10	6
5	11	6	1	40	6
6	11	6	1.5	30	4
7	11	6	2	40	3
8	7	2	0.8	16	2.5
9	7	2	1	20	2
10	5.6	0.6	0.6	6	1

Investment Process

i	$\frac{\bar{R}_i - R_f}{\beta_i}$	$(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(i)}})\beta_i$	$\frac{\beta_i^2}{\sigma^2_{\varepsilon(i)}}$	$\sum_{j=1}^{n}(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(j)}})\beta_j$	$\sum_{j=1}^{n}\frac{\beta_j^2}{\sigma^2_{\varepsilon(j)}}$
1	10	2/10	2/100	44.1/10	87.63/100
2	8	4.5/10	5.63/100	44.1/10	87.63/100
3	7	3.5/10	5/100	44.1/10	87.63/100
4	6	24/10	40/100	44.1/10	87.63/100
5	6	1.5/10	2.5/100	44.1/10	87.63/100
6	4	3/10	7.5/100	44.1/10	87.63/100
7	3	3/10	10/100	44.1/10	87.63/100
8	2.5	1/10	4/100	44.1/10	87.63/100
9	2	1/10	5/100	44.1/10	87.63/100
10	1	0.6/10	6/100	44.1/10	87.63/100

Investment Process

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{n} \left(\frac{\bar{R}_i - R_f}{\sigma_{\varepsilon(i)}^2}\right)\beta_i}{1 + \sigma_m^2 \sum_{i=1}^{n} \frac{\beta_i^2}{\sigma_{\varepsilon(i)}^2}}$$

i	
1	4.52
2	4.52
3	4.52
4	4.52
5	4.52
6	4.52
7	4.52
8	4.52
9	4.52
10	4.52

Investment Process

There is no C* which is maximum. We are considering all the assets (due to SS), hence we have only one value (which we, for our convenience will also denote here by C*). Assets with

$$\frac{\overline{R}_i - R_f}{\beta_i} > C^*$$

will be included in the portfolio and for those $\frac{\overline{R}_i - R_f}{\beta_i} < C^*$

with would be included considering SS

Investment Process

Remember one important thing. The summation is being done from j to n (i.e., all assets you have been provided with).

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{n} \left(\frac{\bar{R}_i - R_f}{\sigma_{\varepsilon(i)}^2}\right)\beta_i}{1 + \sigma_m^2 \sum_{i=1}^{n} \frac{\beta_i^2}{\sigma_{\varepsilon(i)}^2}}$$

Investment Process

$$w_i = \frac{\beta_i}{\sigma^2_{\varepsilon(i)}}\left[\frac{\bar{R}_i - R_f}{\beta_i} - C^*\right]$$

$$C^* = \sigma^2_m \sum_{i=1}^{n} \beta_i wi = \frac{\sigma^2_m \sum_{i=1}^{n} \left(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(i)}}\right)\beta_i}{1 + \sigma^2_m \sum_{i=1}^{n} \frac{\beta_i^2}{\sigma^2_{\varepsilon(i)}}}$$

The n here means the whole set of assets given to you. Here in this example it is 10.

Investment Process

$$w_i = \frac{\beta_i}{\sigma^2_{\varepsilon(i)}}\left[\frac{\bar{R}_i - R_f}{\beta_i} - \frac{\sigma^2_m \sum_{i=1}^{n}\left(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(i)}}\right)\beta_i}{1+\sigma^2_m \sum_{i=1}^{n}\frac{\beta_i^2}{\sigma^2_{\varepsilon(i)}}}\right]$$

i	w(i) (non-normalized wghts)	w*(i) (normalized wghts)
1	0.10965	0.07956
2	0.13060	0.09476
3	0.12414	0.09007
4	0.29654	0.21517
5	0.03707	0.02690
6	-0.02586	-0.01877
7	-0.07586	-0.05505
8	-0.10086	-0.07319
9	-0.12586	-0.09133
10	-0.35173	-0.25521

Investment Process

Using the constant variance-covariance matrix

Here we consider the correlation coefficient between asset i and j as always equal and constant, no matter what is the asset number, i.e., $\sigma_{i,j}=\rho\sigma_i\sigma_j$

The ranking criteria is considered to be the excess return to the standard deviation. i.e.,

$$\frac{\overline{R}_i - R_f}{\sigma_i}$$

Investment Process

Without SS

$$\frac{\bar{R}_i - R_f}{\sigma_i} > C^*$$ include

$$\frac{\bar{R}_i - R_f}{\sigma_i} < C^*$$ do not include

With SS

$$\frac{\bar{R}_i - R_f}{\sigma_i} > C^*$$ include

$$\frac{\bar{R}_i - R_f}{\sigma_i} < C^*$$ include with SS

Investment Process

The problem has the condition

- SS not being allowed
- Risk less lending and borrowing being allowed

Investment Process

The rule works as follows

- Find the excess return to SD for each stock under consideration and rank them from the highest to the lowest
- The optimal consists of investing in all stocks with $\frac{\bar{R}_i - R_f}{\sigma_i} > C^*$
- And not investing in any stock with

$$\frac{\bar{R}_i - R_f}{\sigma_i} \leq C^*$$

Investment Process

We assume $R_f = 5\%$

i	$\bar{R}_i$	$\bar{R}_i - R_f$	σ_i	$\frac{\bar{R}_i - R_f}{\sigma_i}$	$\frac{\rho}{1-\rho+i\rho}\sum_{j=1}^{i}\frac{\bar{R}_j - R_f}{\sigma_j}$		C_i
1	29	24	3	8	½	8	4
2	19	14	2	7	1/3	15	5
3	29	24	4	6	¼	21	**5.25**
4	35	30	6	5	1/5	26	5.2
5	14	9	2	4.5	1/6	30.5	5.08
6	21	16	4	4	1/7	34.5	4.93
7	26	21	6	3.5	1/8	38	4.75
8	14	9	3	3	1/9	41	4.56
9	15	10	5	2	1/10	43	4.3
10	9	4	2	2	1/11	45	4.09
11	11	6	4	1.5	1/12	46.5	3.88
12	8	3	3	1	1/13	47.5	3.65

Investment Process

This C* implies that for all assets above (in the sense $\frac{\bar{R}_i - R_f}{\sigma_i} > C^*$)

will be included in the portfolio. And for assets with $\frac{\bar{R}_i - R_f}{\sigma_i} < C^*$ they

would not be included. This is due to the fact that SS is not being allowed

Investment Process

Remember one important thing. The summation is being done from j=1 to i. This is true for this case as SS is not allowed

Investment Process

$$C^* = \frac{\rho}{1-\rho+n\rho}\sum_{i=1}^{n}\frac{\bar{R}_i - R_f}{\sigma_i}$$

$$w_i = \frac{1}{(1-\rho)\sigma_i}\left[\frac{\bar{R}_i - R_f}{\sigma_i} - C^*\right]$$

This C* is different from the C*′s we have considered under the single index mode. Also the n is different for the case when SS is not allowed and for the case when SS is allowed

Investment Process

Remember one important thing. The summation is being done from j to i. This is true for this case when SS is not being allowed

$$C_1 = \frac{0.5}{1-0.5+1*0.5} * 8 = 4$$

$$C_2 = \frac{0.5}{1-0.5+2*0.5} * [8+7] = 5$$

$$C_3 = \frac{0.5}{1-0.5+3*0.5} * [8+7+6] = 5.25$$

Investment Process

i	w(i) (non-normalized wghts)	w*(i) (normalized wghts)
1	44/24	0.463
2	42/24	0.442
3	9/24	0.095
4		
5		
6		
7		
8		
9		
10		
11		
12		

Investment Process

The problem has the condition

- SS being allowed (normal definition)
- Risk less lending and borrowing being allowed

Investment Process

The rule works as follows

- Find the excess return to SD for each stock under consideration and rank them from the highest to the lowest
- The optimal consists of investing in all stocks with $\frac{\bar{R}_i - R_f}{\sigma_i} > C^*$
- And also investing in any stock (SS) with

$$\frac{\bar{R}_i - R_f}{\sigma_i} \leq C^*$$

Investment Process

We assume $R_f = 5\%$

i	$\bar{R}_i$	$\bar{R}_i - R_f$	σ_i	$\frac{\bar{R}_i - R_f}{\sigma_i}$	$\frac{\rho}{1-\rho+n\rho}\sum_{j=1}^{n}\frac{\bar{R}_j - R_f}{\sigma_j}$		C_i
1	29	24	3	8	1/13	47.5	3.65
2	19	14	2	7	1/13	47.5	3.65
3	29	24	4	6	1/13	47.5	3.65
4	35	30	6	5	1/13	47.5	3.65
5	14	9	2	4.5	1/13	47.5	3.65
6	21	16	4	4	1/13	47.5	3.65
7	26	21	6	3.5	1/13	47.5	3.65
8	14	9	3	3	1/13	47.5	3.65
9	15	10	5	2	1/13	47.5	3.65
10	9	4	2	2	1/13	47.5	3.65
11	11	6	4	1.5	1/13	47.5	3.65
12	8	3	3	1	1/13	47.5	3.65

Investment Process

There is no C* which is maximum. We are considering all the assets (due to SS), hence we have only one value (which we, for our convenience will also denote here by C*). Assets with

$$\frac{\overline{R}_i - R_f}{\sigma_i} > C^*$$

will be included in the portfolio and for those $\frac{\overline{R}_i - R_f}{\sigma_i} < C^*$

with would be included considering SS

Investment Process

Remember one important thing. The summation is being done from j=1 to n. This is true for this case as SS is allowed

Investment Process

$$C^* = \frac{\rho}{1-\rho+n\rho}\sum_{i=1}^{n}\frac{\bar{R}_i - R_f}{\sigma_i}$$

$$w_i = \frac{1}{(1-\rho)\sigma_i}\left[\frac{\bar{R}_i - R_f}{\sigma_i} - C^*\right]$$

This C* is different from the C*′s we have considered under the single index mode. Also the n is different for the case when SS is not allowed and for the case when SS is allowed

Investment Process

i	w(i) (non-normalized wghts)	w*(i) (normalized wghts)
1	1.45	0.88
2	1.68	1.02
3	0.59	0.36
4	0.23	0.14
5	0.43	0.26
6	0.09	0.06
7	-0.03	-0.02
8	-0.22	-0.13
9	-0.33	-0.21
10	-0.83	-0.51
11	-0.54	-0.33
12	-0.88	-0.54

Investment Process

The problem has the condition

- SS being allowed (Lintner's definition)
- Risk less lending and borrowing being allowed

Investment Process

The rule works as follows

- Find the excess return to SD for each stock under consideration and rank them from the highest to the lowest
- The optimal consists of investing in all stocks with $\frac{\bar{R}_i - R_f}{\sigma_i} > C^*$
- And also investing in any stock (SS) with

$$\frac{\bar{R}_i - R_f}{\sigma_i} \leq C^*$$

Investment Process

There is no C* which is maximum. We are considering all the assets (due to SS), hence we have only one value (which we, for our convenience will also denote here by C*). Assets with

$$\frac{\overline{R}_i - R_f}{\sigma_i} > C^*$$

will be included in the portfolio and for those $\frac{\overline{R}_i - R_f}{\sigma_i} < C^*$

with would be included considering SS

Investment Process

We assume $R_f = 5\%$

i	$\bar{R}_i$	$\bar{R}_i - R_f$	σ_i	$\frac{\bar{R}_i - R_f}{\sigma_i}$	$\frac{\rho}{1-\rho+n\rho}\sum_{j=1}^{n}\frac{\bar{R}_j - R_f}{\sigma_j}$		C_i
1	29	24	3	8	1/13	47.5	3.65
2	19	14	2	7	1/13	47.5	3.65
3	29	24	4	6	1/13	47.5	3.65
4	35	30	6	5	1/13	47.5	3.65
5	14	9	2	4.5	1/13	47.5	3.65
6	21	16	4	4	1/13	47.5	3.65
7	26	21	6	3.5	1/13	47.5	3.65
8	14	9	3	3	1/13	47.5	3.65
9	15	10	5	2	1/13	47.5	3.65
10	9	4	2	2	1/13	47.5	3.65
11	11	6	4	1.5	1/13	47.5	3.65
12	8	3	3	1	1/13	47.5	3.65

Investment Process

Remember one important thing. The summation is being done from j=1 to n. This is true for this case as SS is allowed

Investment Process

$$C^* = \frac{\rho}{1-\rho+n\rho}\sum_{i=1}^{n}\frac{\bar{R}_i - R_f}{\sigma_i}$$

$$w_i = \frac{1}{(1-\rho)\sigma_i}\left[\frac{\bar{R}_i - R_f}{\sigma_i} - C^*\right]$$

This C* is different from the C*′s we have considered under the single index mode. Also the n is different for the case when SS is not allowed and for the case when SS is allowed

Investment Process

i	w(i) (non-normalized wghts)	w*(i) (normalized wghts)
1	1.45	0.20
2	1.68	0.23
3	0.59	0.08
4	0.23	0.03
5	0.43	0.06
6	0.09	0.01
7	-0.03	-0.01
8	-0.22	-0.03
9	-0.33	-0.05
10	-0.83	-0.11
11	-0.54	-0.07
12	-0.88	-0.12

Investment Process

Capital Asset Pricing Model (CAPM)

We are mainly concerned with two important problems

- To determine the best course of action in the investment process, i.e., how to device the portfolio.
- To determine the correct, arbitrage free or fair or equilibrium price of any asset/security (we will be mainly concerned with this question)

Investment Process

In answering the second question we come across the CAPM which was developed by Sharpe, Lintner and Gossain

Investment Process

Assumptions

1) No transaction costs
2) Assets are infinitely divisible
3) Absence of personal tax
4) Investors transaction cannot affect the price of an asset
5) Investors make decision based on return and risk criterion

Investment Process

Assumptions

6) Unlimited SS is allowed
7) Unlimited risk less lending and borrowing is allowed
8) Investors define relevant period in exactly the same manner
9) Investors have identical expectations with respect to the necessary inputs for the portfolio decision
10) All assets are marketable

Investment Process

For the one fund theorem which we had studied, a natural question arises what would be the fund Q (risk assets) which everyone would purchase. Is it the market index or something else.

Investment Process

That fund is the market index portfolio, i.e., it is the fund that contains share of every stock in proportion to the stock′s representation in the entire market. An asset′s weight is defined as the proportion of the portfolio capital that is allocated to the asset.

Investment Process

Hence the weight of the asset in the market portfolio is equal to the proportion of that asset′s total capital value to the total market capital value. This weight is called the market capitalization weights and is denoted by w_i, as we already have seen.

Investment Process

Capital market/Pricing line

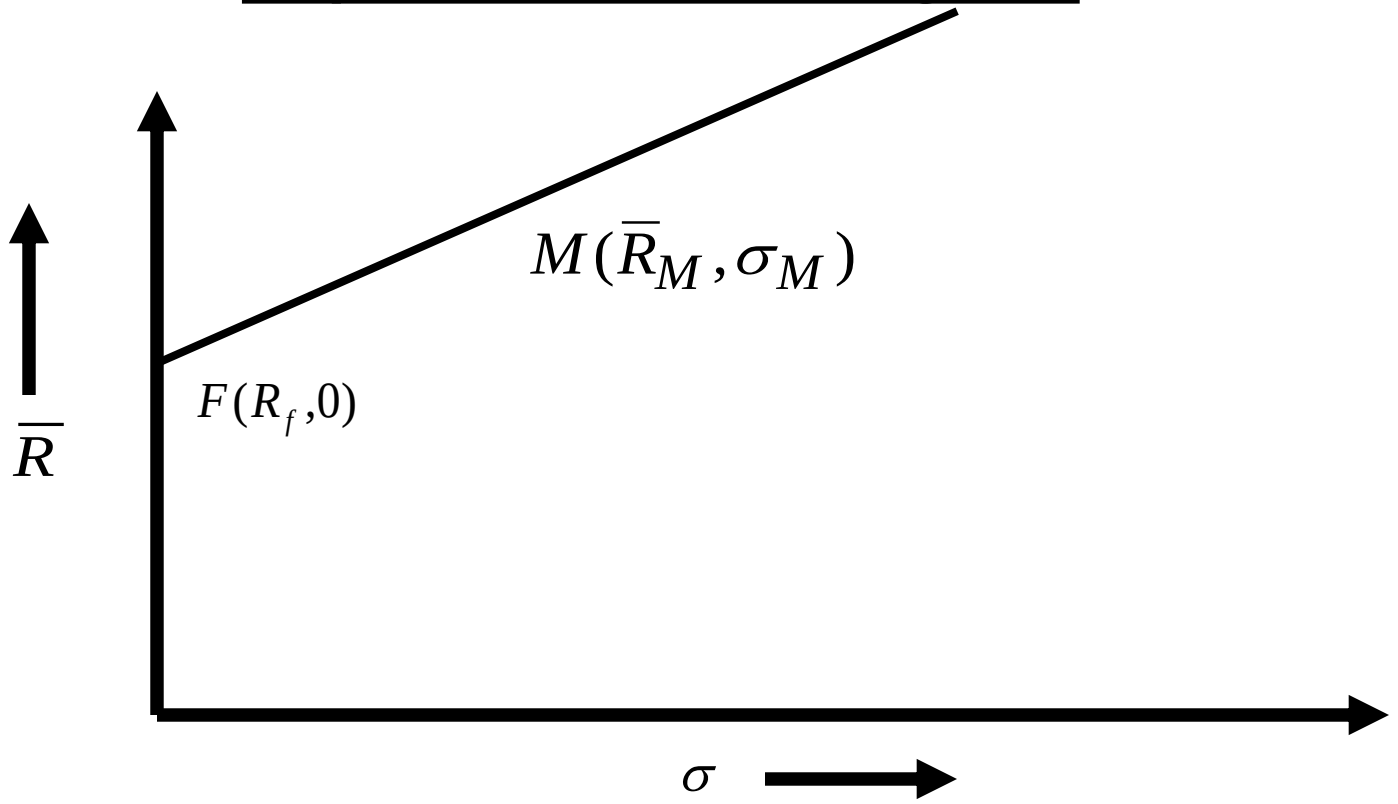
The capital market/pricing line shows the relation between the expected rate of return and the risk. Thus as risk increases so does the expected return. Mathematically we can define this as a linear regression, given by

$$\overline{R}_{P[efficient(i)]} = R_f + \frac{\overline{R}_M - R_f}{\sigma_M}\sigma_{P[efficient(i)]}$$

The slope of the capital market/pricing line is the price of risk

Investment Process

Capital market/Pricing line



Investment Process

The capital market/pricing line relates the expected rate of return of an efficient portfolio to its standard deviation. Remember it does not show how the expected rate of return of an individual asset is related to its corresponding risk.

Investment Process

CAPM

If the market portfolio M is efficient, then the expected return for any efficient portfolio is given by

$$\bar{R}_{P[efficient(i)]} - R_f = \beta_{P[efficient(i)]}(\bar{R}_M - R_f)$$

$$\bar{R}_{P[efficient(i)]} - R_f = \frac{\sigma_{P[efficient(i)],M}}{\sigma_M^2}(\bar{R}_M - R_f)$$

Investment Process

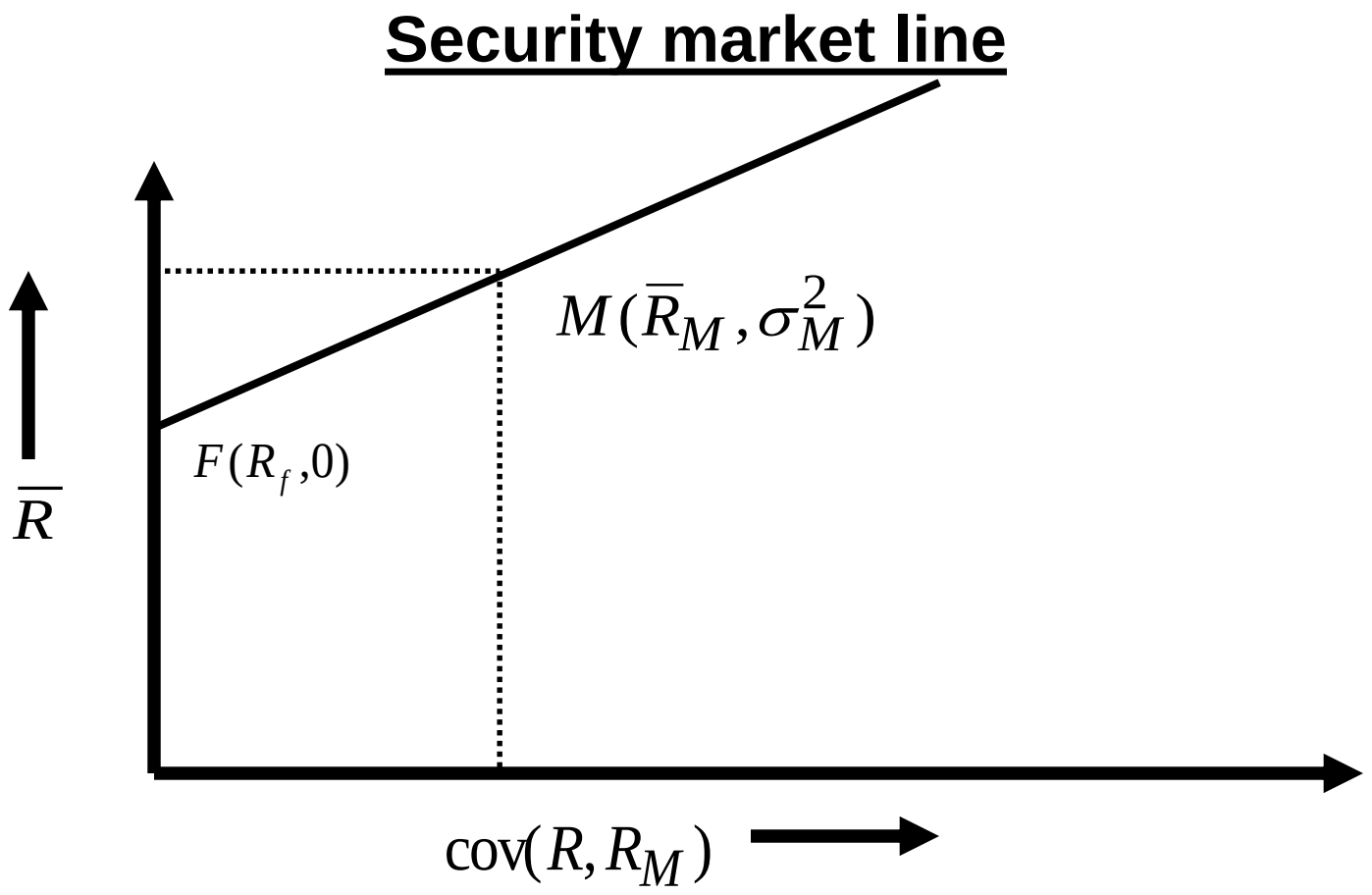
Beta is the normalized version of the covariance of the asset with the market portfolio. Hence we can say that the CAPM formula states that the expected excess return of an asset is directly proportional to its covariance with the market. It is this covariance that determines the expected excess rate of return. CAPM changes our concept of the risk of an asset from that of σ to β.

Investment Process

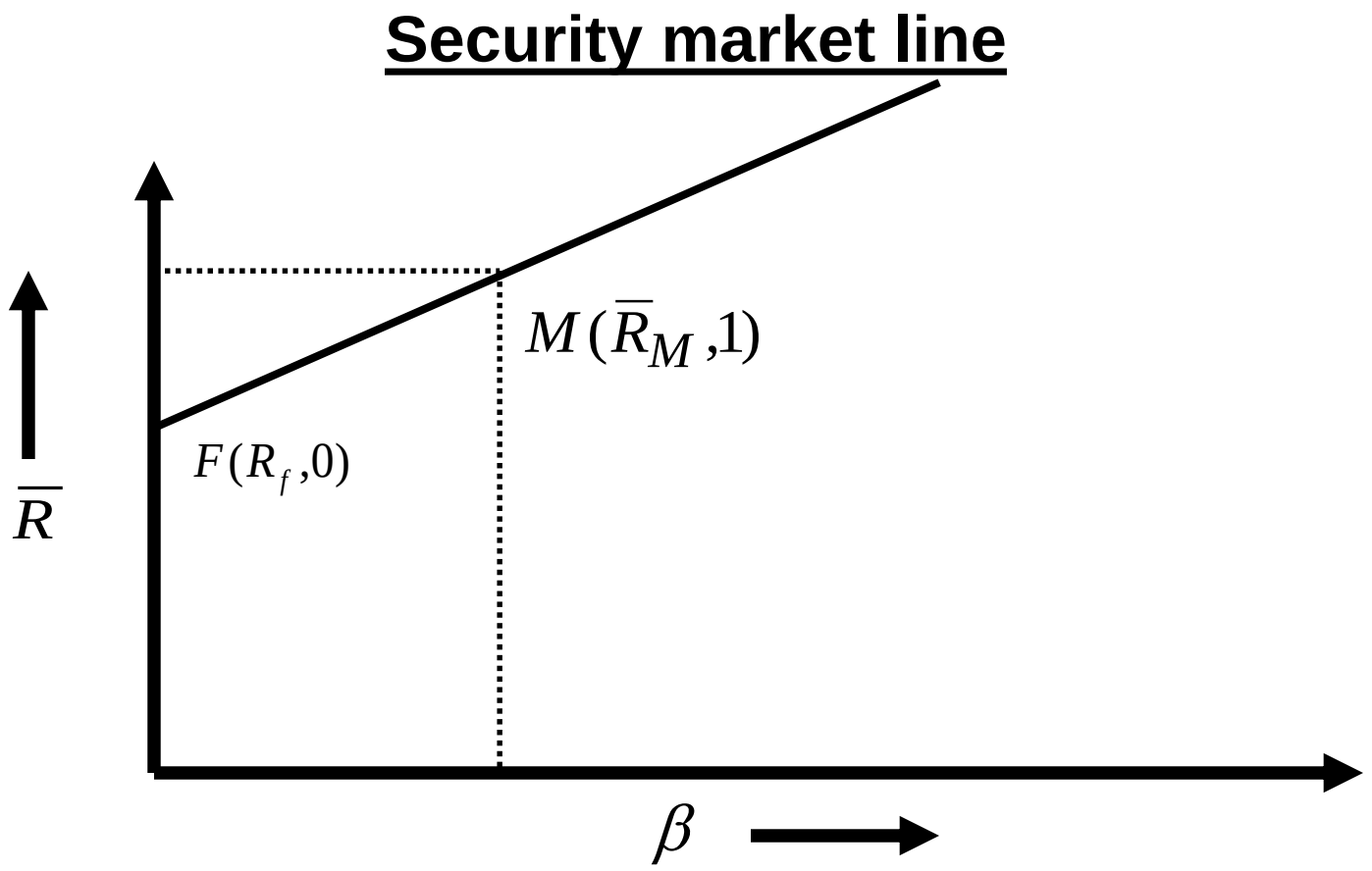
Security market line

The CAPM can be expressed in graphical form by regarding the formula as a linear relationship. This relationship is termed as the security market line (as shown in the next slides)

Investment Process



Investment Process



Investment Process

In both the graphs we have different variables in the x-axis. Remember the essence of CAPM emphasizes that any asset/portfolio should fall on the security market line. Thus this line expresses the risk reward structure of an asset/portfolio according to the CAPM and emphasizes that the risk of an asset/portolio is a function of its covariance with the market or equivalently a function of its beta

Investment Process

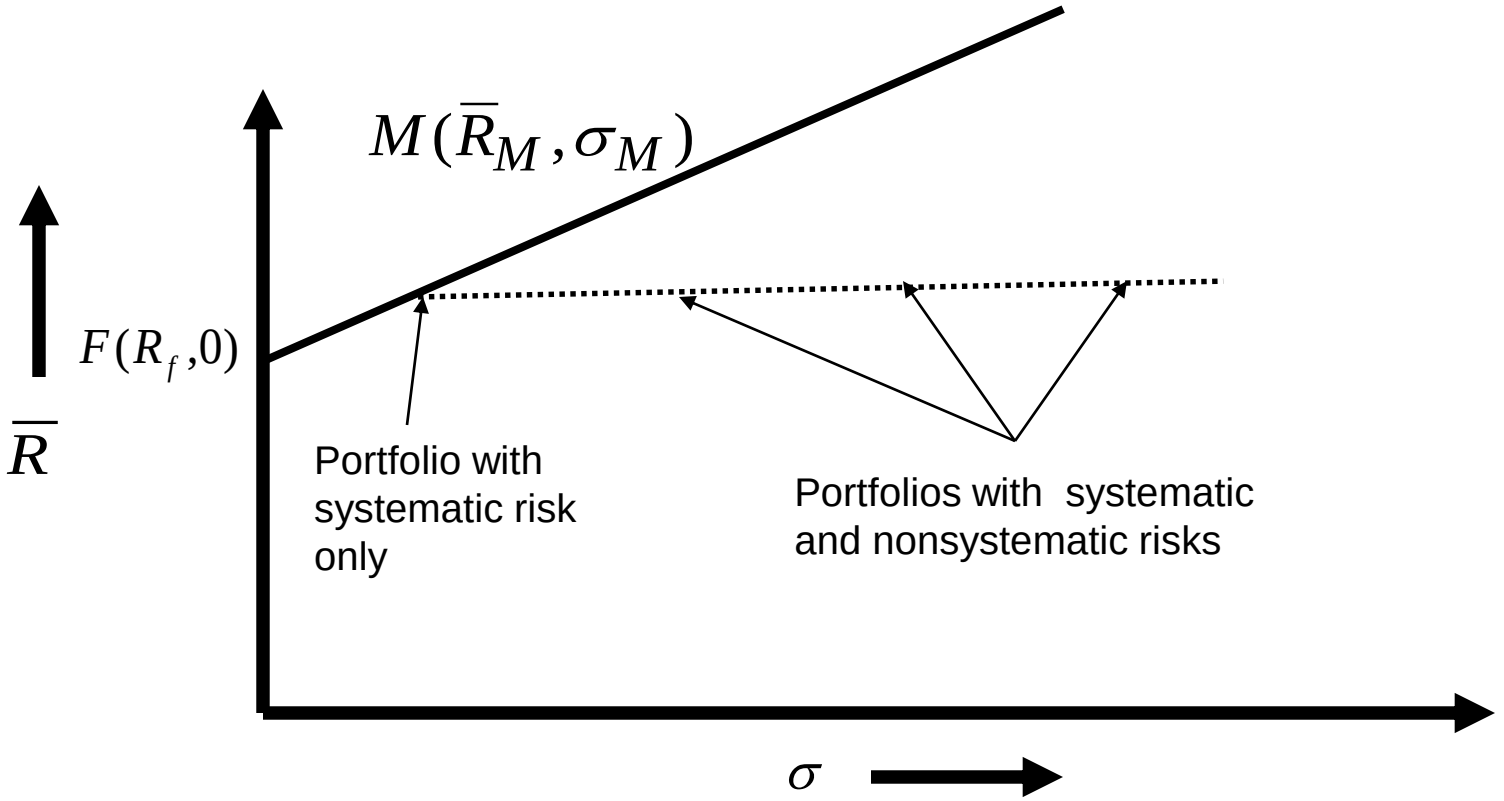
We know the systematic risk is given by $\beta_i\sigma^2_M$ and the nonsystematic risk as $\sigma^2_{\varepsilon(i)}$.

The systematic risk is the risk associated with the market and cannot be reduced with diversification, while nonsystematic risk can be made zero with diversification.

Investment Process

Consider an asset on the capital market line and in that case it would be an efficient portfolio which is formed by investing some proportion in the market portfolio and the rest in the risk free interest rate. If nonsystematic risk is zero then the portfolio is exactly on the capital market line. So more the nonsystematic risk greater is the shift towards the right.

Investment Process



Investment Process

For performance evaluation we have the CAPM model as

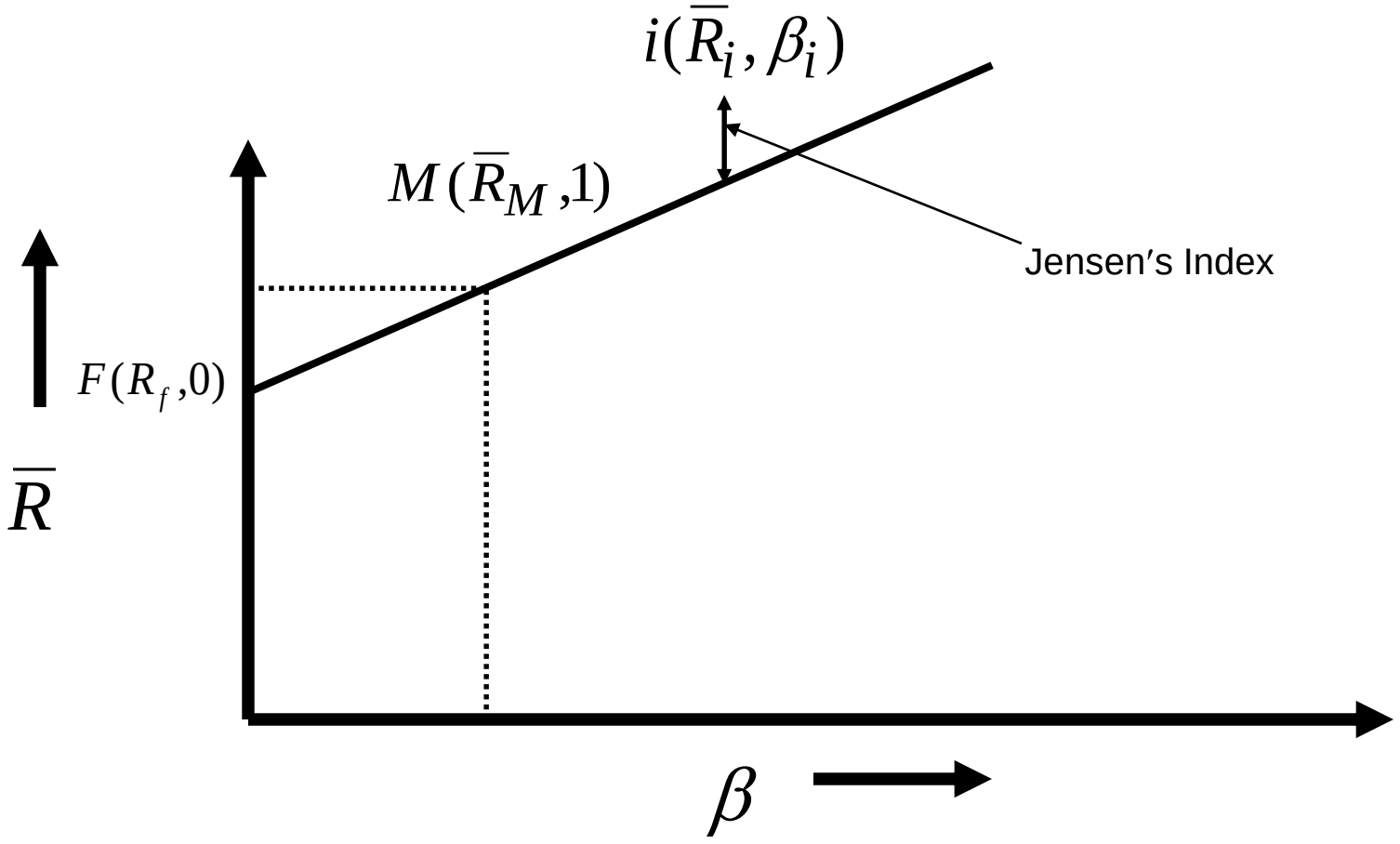
$$\hat{\bar{R}}_i - R_f = \hat{\beta}_i(\hat{\bar{R}}_M - R_f) + J$$

J is the error term and is the Jensen′s index

Investment Process

According to CAPM, the value of J should be zero when the true expected returns are used. Hence J measures approximately how much the performance of any index/stock has deviated from the theoretical value of zero. A positive J implies the stock did better and a negative J implies it did worse. The value of J tells us nothing about the fund, but instead it is a measure of the volatility of the CAPM. If CAPM is valid then every security/fund must satisfy CAPM exactly. If we find a **so called** efficient portfolio security with non zero J then the market is **itself inefficient**.

Investment Process



Investment Process

In order to measure the efficiency of a security or the market we use, S, the Sharpe index and the formula is given by $\hat{\bar{R}}_i - R_f = S\hat{\sigma}_i$

Investment Process

